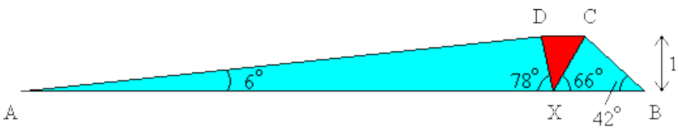
math news

Vol. XXVII, No. 2 November, 2013

Solution to our Previous Puzzle

In trapezoid ABCD shown, with sides AB and CD parallel, angle DAB = 6^0 and angle ABC = 42^0. Point X on side AB is such that angle AXD = 78^0 and angle CXB = 66^0. If AB and CD are 1 inch apart, it can be shown as follows that AD + DX – (BC + CX) = 8 inches.



Dropping a perpendicular (of length 1) from D to AX, and similarly from C to BX, we see that AD = cosec 6^o, DX = cosec 78^o, BC = cosec 42^o, and CX = cosec 66^o. So, we're to prove that cosec 6° + cosec 78° − cosec 42° − cosec 66° = 8. Notice that, for x = 6°, 78°, −42°, −66°, and 30°, sin 5x = ½. We now express sin 5x in terms of sin x. De Moivre's theorem states that for any real number x and any integer n, $\cos nx + i \sin nx = (\cos x + i \sin x)^n$. Setting n = 5, expanding the right-hand side using the binomial theorem, and equating imaginary parts, we obtain

$$\begin{aligned}\sin 5x &= \sin^5 x - 10 \sin^3 x \cos^2 x + 5 \sin x \cos^4 x \\ &= \sin^5 x - 10 \sin^3 x (1 - \sin^2 x) + 5 \sin x (1 - \sin^2 x)^2 \\ &= 16 \sin^5 x - 20 \sin^3 x + 5 \sin x\end{aligned}$$

This result can also be obtained by means of trig identities. Setting s = sin x, it follows that the five distinct numbers sin 6°, sin 78°, −sin 42°, −sin 66°, and sin 30° = ½, (1) are roots of the equation $16s^5 - 20s^3 + 5s = ½$, or, equivalently, of $32s^5 - 40s^3 + 10s - 1 = 0$. (2) By the Fundamental Theorem of Algebra, (2) has exactly five roots, up to multiplicity, and hence these must be precisely the *distinct* roots identified in (1).

Since s = ½ is a root of (2), the equation factorizes: $(2s - 1)(16s^4 + 8s^3 - 16s^2 - 8s + 1) = 0$, yielding the quartic equation whose roots are sin 6°, sin 78°, −sin 42°, and −sin 66°. As s = 0 is not a root of this quartic equation, we may divide by s^4, and, setting t = 1/s, obtain $t^4 - 8t^3 - 16t^2 + 8t + 16 = 0$, an equation whose roots are cosec 6°, cosec 78°, −cosec 42°, and −cosec 66°.

By Viète's formulas, the sum of these roots is 8. Thus, AD + DX − (BC + CX) = 8 inches.

Probably Some Statistics

When the mean, median, and mode of the list

10, 2, 5, 2, 4, 2, x

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

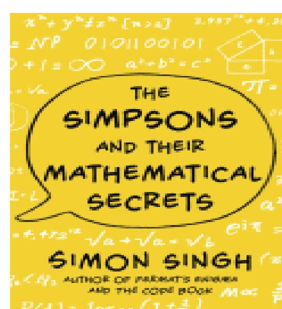
Spring '14 Upper-Level Course Offerings

course		*days*	*time*	*instructor*
236.001	Calculus I	MTRF	8:00	Revennaugh
236.002	Calculus I	MTRF	11:00	Forsythe
237.001	Calculus II	MTRF	11:00	Barnet
237.002	Calculus II	MTRF	2:00	Barnet
238.001	Calculus III	MTRF	2:00	Hughes
380.001	Int. Prob. & Stat.	MWF	1:00	Forsythe
426.001	Complex An.	MWF	12:00	Hughes
432.001	Diff. Equations	MWF	1:00	Dunmyre
437.001	Comb. & Gr. Th.	TR	3:30	Lemmert
470.001	Math. Modeling	MWF	10:00	Barnet
480.001	Prob. & Stat.	MWF	11:00	Hegde

FSU Honors Program

Mathematics majors are encouraged to consider participating in FSU's honors program. Ten honors courses will be offered in Spring '14 (not counting thesis) and one in Intersession '14. See http://www.frostburg.edu/dept/honr/ and your advisor for more information.

Mathematics --- Ay Caramba!



Cleverly embedded in many plots of The Simpsons are subtle references to mathematics, ranging from well-known equations to cutting-edge theorems and conjectures. *The Simpsons and Their Mathematical Secrets*, by Simon Singh, is published by Bloomsbury Publishing PLC, 2013.

Singh also wrote *Fermat's Enigma* and *The Code Book*.

KME Corner

Kappa Mu Epsilon will have a table at the Majors Fair on Wednesday, November 13th, from 11:00 until 2:00. All students are encouraged to stop by.